ANALYSIS OF MURGUE'S THEORY

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ANALYSIS OF MURGUE'S THEORY

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CENTRIFUGAL BLOWERS AND PUMPS

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Analysis of Murgue's Theory
For Centrifugal Blowers and Pumps.

Introduction.

The main object of the thesis is to analyze the theory of the centrifugal pump, and to discuss the varying opinions in regard to the maximum height of the fluid column "H" which the ideal pump would support when revolving at a given speed with a velocity "V" at the tips of the fan or impeller blades.

In one of the references on the subject, the author, in replying to the contrary, makes a statement that after all it makes no difference whether the theoretical height of delivery is

$$\frac{V^2}{2g}, \text{ or } \frac{V^2}{g}$$

as long as we have to use a coefficient which must be determined by experiments for each case, so that the actual value will not suffer

from the change of the theoretical formula.

This consideration is not right. One will say, for instance, "What is the use of the Carnot or Rankin cycle for the steam engine as long as we have the actual thermal efficiency of the machine; that is all we care for in practice." But the idea in such consideration is this: If we see that our machine approaches very near the theoretical limit we do not care to improve it any more at the expense of some practical form of energy. The same with the pump; If the height of delivery is some function of V, it is very important to find the value of that function for ideal conditions, otherwise we may keep on trying to improve our pumps past the theoretical limit.

Since the value of "H" is always calculated from the density of the fluid existing inside the discharge opening of the pump, we are right in assuming that all the theoretical considerations will apply in the same degree to the air blower as to the water pump, and the two, therefore, should not be differentiated.

The authorities on the subject, however, disagree openly in the case of the water pump, some insisting that the theoretical value of "H" is $\frac{V^2}{2g}$ and others trying to prove that it is $\frac{V^2}{g}$, but none of them advances any contradiction with respect to the theory of the air blower, or Murgne's Theory, which is given as the original proof that the initial depression in a centrifugal blower is theoretically $\frac{V^2}{g}$

It is therefore considered best, in order to bring the subject to a clear understanding, to quote some of the authorities and to analyze their theories according to the given assumptions.

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Prof. Church: "Hydraulic Motors", p. 175, derives an equation for the theoretical "speed of impending delivery" for a centrifugal pump of the usual volute construction to be

$$V_n = (\sqrt{2gh}) \div (\sqrt{1-(\tfrac{r_1}{r_2})^2}),$$

from which

$$h = \sqrt{1-(\tfrac{r_1}{r_2})^2} \cdot \frac{V^2}{2g},$$

where h is the maximum head, r and r the inner and outer radii of the blades. According to

this equation, if we do not neglect the small value of $\left(\frac{r_1}{r_2}\right)^2$ we have h, theoretically, less than $\frac{V^2}{2g}$

On the next page of the same work results of experiments are quoted and one of them shows a value for h much greater than $\frac{V^2}{2g}$. A little further the same author says that after the pump is once started the velocity of the tips may sometimes be allowed to sink below the theoretical value. Prof. Church gives no further explanation for this fact.

The same theoretical value for the maximum head is given in a paper read by Mr. Wm. D. Weber, A.S.M.E. 9-233, where, among other things quoted from the Hon. Mr. Parsons, it says, "A fan when rotating will support a column of water the velocity due to whose height is equal to the tangential velocity of the circumference of the fan".

In another paper given by Mr. W. B. Gregory, A.S.M.E., 22-262, this question is brought up openly, Mr. Gregory derives the

theoretical value of the maximum head to be $\boxed{\frac{V^2}{2g}}$, while the results of his tests given in the same paper have a maximum value for $H = 1.24 \frac{V^2}{2g}$. Prof. Wm. Kent in opposition uses this fact to show that the "time honored formula" is wrong, since it gives a value less than does actual practice, and therefore the theoretical value for H must be $\frac{V^2}{g}$

But strictly speaking, if $\frac{V^2}{2g}$ is wrong it does not follow that $\frac{V^2}{g}$ is right, and there is no danger in having a coefficient to a theoretical formula in practice larger than unity when that coefficient does not indicate the efficiency of the machine. The comparison made by Mr. Kent to a jet of water striking a plane surface where the pressure becomes twice the velocity head, cannot be applied to the condition of zero discharge, because the velocity is tangent to the surface and has no normal component.

The original theory of centrifugal fans developed by Daniel Murgue in 1872 as

given by Prof. Carpenter in his work on "Heating and Ventilating", (Fourth edition, p. 350) will be reproduced here in order to analyze the treatment and see whether the proof given is sufficient to show that, theoretically,

$$H = \frac{V^2}{g}$$

This theory is reproduced also in Sibley Journal of Engineering, Nov. 1902, p. 44, as follows: "In the determination of the initial depression the ideal ventilator only is considered, in which there is no loss by friction or shock. Two cases are considered; first, when the suction wheel or exhaust fan is revolving without a casing, and, second, when the wheel is provided with the Guibel casing and chimney. Denote the radius of the inlet by r; that of the external circumference by R; the angular speed of ratation in feet per second by w; the absolute speed of the extremities of the vanes by R; the tangential speed by U".

"Suppose that the air before entering

the fan is motionless and that it traverses the inlet with the speed of V_o we should then have a negative head corresponding to the pressure

$$-\frac{V_o^2}{2g} \qquad \dots\dots\dots (6)$$

"The interval between two consecutive vanes forms an evasee canal which the air enters with a certain speed V_1 and leaves with a less speed, V_2. This slowing action produces, according to Bernouill's theory, a gain of pressure expressed by the difference

$$\frac{V_1^2}{2g} - \frac{V_2^2}{2g} \qquad \dots\dots\dots\dots (7)$$

"The speed of entry V_1, is the resultant of the two speeds, the one V_o being radial, the other, ωr, being tangential and if there is no shock or loss,

$$V_1^2 = V_o^2 + \omega^2 r^2$$

"Substituting this value of V_1, in the preceding equation we have

$$\frac{V_o^2 + \omega^2 r^2}{2g} - \frac{V_2^2}{2g} \qquad \dots\dots\dots\dots (8)$$

"Supposing a prismatic element of the air passing through the fan at a distance X

from the center, having a radial height dX and a base s in a perpendicular direction, and a density δ, we shall have for the mass of this element

$$\frac{\delta s dX}{g} \ldots\ldots (9)$$

"The centrifugal force developed by its rotation

$$dF = \frac{\delta s dX}{g}\omega^2 X \ldots\ldots (10)$$

"By dividing the above by δs we have the pressure expressed per unit of area in the column of air as follows:

$$dh = \frac{\omega^2 X dX}{g} \ldots\ldots (11)$$

"Integrating this last between the limits $X = r$ and $X = R$, we shall have a total difference of pressure from the inlet to the outside of the vanes,

$$h = \frac{\omega^2 R^2 - \omega^2 r^2}{2g} \ldots\ldots (12)$$

"Adding the results in equation (6), (8) and (12) together and substituting $U = \omega R$ we have the initial depression.

$$H = \frac{U^2}{2g} - \frac{V^2}{2g} \ldots\ldots (9)$$

"The above formula is general" and so on. The effect of the chimney is explained by the theory Bernouills', thus; If

V' is the speed of the air at the bottom of the Guibal chimney, and W is the speed at the top, the increase in pressure would be

$$\frac{V'^2}{2g} - \frac{w^2}{2g} \cdots\cdots\cdots (14)$$

"The speed V is that of the air on leaving the vanes, and it is thus the resultant of the tangential speed u, and of the delivery V_2. These last two quantities form a parallelogram of velocities from which if α be the angle at which the vanes strike the exterior circumference,

$$V'^2 = U^2 - V_2^2 - 2UV_2 \cos\alpha \cdots\cdots\cdots (15)$$

"Substituting this value of V'^2 in equation (14) we have the total pressure produced by the chimney. Adding this result to equation (13), which gave the pressure due to the fan alone, we have the total depression produced by the fan and chimney as follows:

$$H_1 = \frac{U^2}{g} - \frac{UV_2\cos\alpha}{g} - \frac{w^2}{2g} \cdots\cdots\cdots (16)$$

"This equation is a maximum when α is 90 degrees and $\cos\alpha$ zero, from which it appears that the highest theoretical efficiency should be produced when the tips of the blades are radial. If $\alpha = 90$ degrees we have $H_1 = \frac{U^2}{g}$

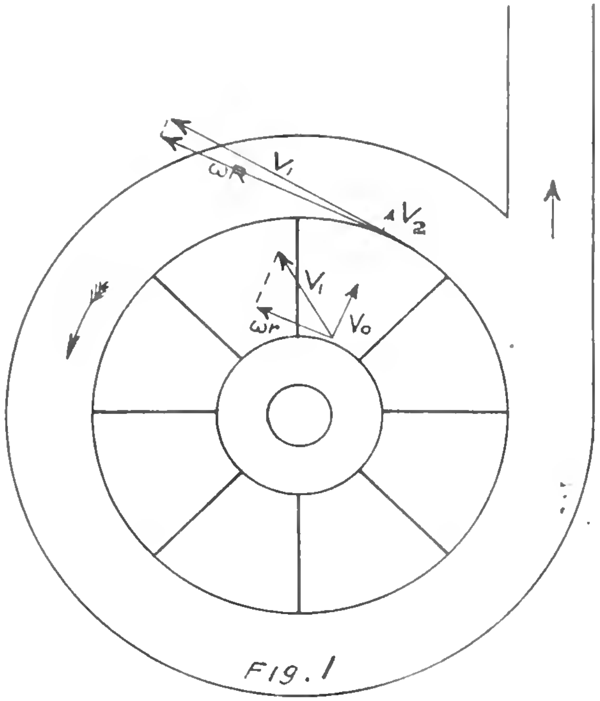

Fig. 1

Fig. 1 shows a diagramatic representation of the quantities involved in the previous theory. This figure is not given in the quoted article, but it was laid out by the writer according to the statements given in the article in order to show more clearly and to discuss the meaning of those statements.

Equation (7), based on Bernouill's

theory for a moving fluid, credits the system with a gain of pressure for the reduction of speed from V_1 to V_2 while passing through the region of the vanes. As given by the same statement, and shown on the figure here, V_1 the speed of entry is an absolute speed resulting from V_0 the radial and ωr the tangential velocity, But V_2 is the radial speed at the leaving, ωR is the tangential at the same place, and V' the one used in the latter part of the article, is the resultant of those two. Now the way the statement preceding equation (7) is made it does not appear to be mathematically right, first, because the component V_2 should not be substracted from the resultant V_1 and second, if there is some radial velocity V_2 at the leaving of the vanes, then the absolute velocity is V' and the credit for the change in speed must be

$$\frac{V_1^2}{2g} - \frac{V'^2}{2g}$$

and therefore the result given in the equation (13) as a general formula must be

$$H = \frac{U^2}{2g} - \frac{V'^2}{2g}$$

and not

$$H = \frac{U^2}{2g} - \frac{V^2}{2g}$$

The reason for this discrepancy is that Bernouillis' theory for a rotating channel with a forced vertex applies only to the radial velocity of the fluid, or, more generally, to the motion of the fluid relative to the blades, while the increase in tangential velocity has to be credited to the system as an additional amount of energy supplied by an outside medium increasing the total (potential and dynamic) head of the fluid. Applying Bernouillis' theory for a steady flow between the entry to the fan and escape from the blades we have

$$\frac{P_1}{\gamma} + b + \frac{V_o^2}{2g} + \frac{\omega^2 R^2}{2g} = \frac{P_2}{\gamma} + b + \frac{V_2^2}{2g} ,$$

$\frac{P_1}{\gamma}$ is the hydro static pressure at the inlet, V_o is the velocity of the fluid at the same point, and according to the assumptions made in the quoted article the sum of the two is zero, $\frac{\omega^2 R^2}{2g}$ is the increase of pressure between the two points due to the centrifugal force of the revolving mass.

Solving for $\frac{P_2}{y}$ or "H" the hydrostatic pressure at the escape of the blades we have the general equation as given in the quoted theory

$$\frac{P_2}{y} = H = \frac{\omega^2 R^2}{2g} - \frac{V_2^2}{2g} = \frac{U^2}{2g} - \frac{V_2^2}{2g}$$

Prof. Carpenter, giving his own theory of the centrifugal fan blower (Heating and Ventilating, p. 359) says that the maximum pressure produced by a fan or blower corresponds to the initial depression in Murgue's theory. The question is, does it correspond or not. Experiment shows that with a properly designed fan the pressure may increase considerably after the discharge has started. Prof. Carpenter goes on to say: "This pressure is obtained only when the work imparted to the fan is all utilized in overcoming resistances, as, for instance, in a pressure fan when the discharge opening is entirely closed. For this case, if there is no loss of energy due to eddies or other resistances, we shall find, since the work done is equal to the weight moved or the pressure H overcome in one second multiplied by the

space, that

$$H(\tfrac{1}{2}U) = \frac{wU^2}{2g}"$$

(Where H is the maximum pressure in the peripheral velocity of the fan wheel, W = weight of air moved per second). This equation is not at all clear, since, when the discharge opening of the fan is entirely closed, W is manifestly zero, and the work required to maintain uniform rotation is none except friction and eddy current resistances.

Mr. Blaine in his work "Hydraulic Machinery", p. 113, gives a similar proof for the relation $H = \frac{V^2}{g}$ by analyzing the interchange of energy in the pump. But that proof in the way that it is given, shows rather the opposite that $H = \frac{V^2}{2g}$. Mr Blaine gives the following figure and says:

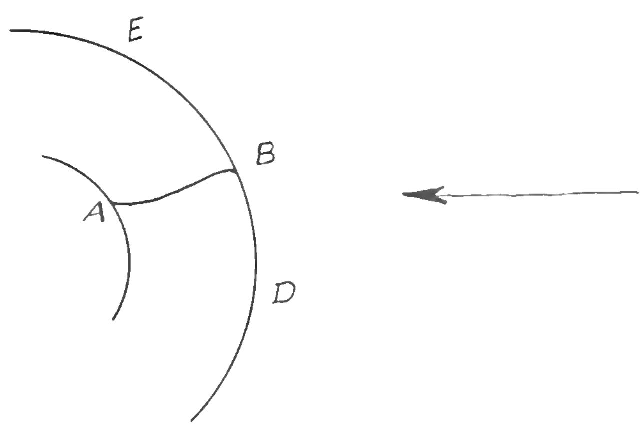

Fig. 89

"Take a simple case (Fig. 89); the water had no momentum in the direction of the motion of the wheel before it entered it at A; having entered it now moves along the vane AB gradually attaining the velocity of the wheel, and then it finally leaves at B. If the vane is radial at B it has the same velocity as the wheel just before it leaves. Let this velocity be V feet per second. Then every pound of water leaving B leaves with a tangential momentum $\frac{V}{32.2}$, and retards the wheel with a force of this amount acting as B. This force times V is the energy the one pound of water receives per second from the wheel which is equal to $\frac{V^2}{32.2}$ and so on".

Of course if a pound of water is given an amount of energy equal to $\frac{V^2}{g}$ it will certainly rise to a height H equal to $\frac{V^2}{g}$. It is also true that every pound of water in this case leaves B with a tangential momentum $\frac{V}{g}$. But is it true that the retarding force which is of the same amount acts totally at B as the author says. On the contrary it is more natural to admit that the

greatest effort is exerted at A where the greatest change of momentum occurs while at B that change is only $\Delta V = \omega \Delta r$ where ω is the angular velocity of the wheel, and Δr is the increment in distance from the center along the line AB. Now, since the force acts through the whole process, beginning practically at v = zero and ending at v = v, therefore the total energy is the force MV times the average velocity $\frac{V}{2}$ or a total amount of $\frac{V^2}{2g}$ per pound and not $\frac{V^2}{g}$

A very elaborate investigation of centrifugal pumps is given by Mr. C. B. Stewart in bulletin No. 173 of the University of Wisconsin. A series of tests are shown of a 6" vertical centrifugal pump, and a complete discussion of the theory in comparison with practical results is given there. Mr. Stewart (p. 539-548)derives the equation for the theoretical head of the centrifugal pump

$$H = \frac{V^2}{2g}$$

using both methods; Bernouillis' theory with

centrifugal force of rotation, and also the kinetic energy method. Mr. Stewart does not differentiate between the maximum head and the head of impending delivery. The assumptions are made, however, for a steady radial flow distributed over the whole circumferential area of the impeller. For these assumptions both methods are given in a plain and obvious way, and the proof is sufficient that the theoretical <u>maximum</u> head shall be $\frac{V^2}{g}$. But here comes in a contradiction noticed by Mr. Stewart himself. On page 548 of the same bulletin we read: "Equations (9) to (12) (these equations are different forms of $H=\frac{V^2}{g}$) have been used as the standard equations for theoretical head developed by the impeller of a centrifugal pump. Experiment seems to show that they are true for impending delivery, but as soon as flow begins they do not apply", and so on. Now if the equations do not hold true for the assumed conditions of steady flow on which the whole proof was based they cannot be applied for impending delivery when there is no flow, and the condition is contrary to the assumption.

On page 554 of the same bulletin Mr. Stewart makes a statement which seems to be contradictory to the previous quotation, but at the same time it shows clearly that the contradiction comes from the interchange of the two terms, the maximum head and the head of impending delivery. The statement is "Equation (19) shows that the theoretical head H for impending delivery is equal to

$$\frac{V^2}{g}$$

while experiment shows that for impending delivery the maximum actual head possible approaches

$$\frac{V^2}{2g}$$

as a limit. It is seen from this that the velocity head

$$\frac{V^2}{2g}$$

is entirely lost for impending delivery, the head utilized being very nearly the entire centrifugal head

$$\frac{V^2}{2g}$$

"As the flow commences from the condition of impending delivery; each pound of water discharged will possess the kinetic energy

$$\frac{V^2}{2g}$$

in addition to its pressure energy", and so on.

This is exactly what we want to make clear, that in the centrifugal pump, where the working fluid is incompressible, the head of impending delivery is not necessarily the maximum head that can be obtained by the given speed of the blades. There is no marked difference between the two in the air blower provided with a Guibal casing and chimney, because the air being a compressible fluid will continue to flow through the vanes during compression and may build up the maximum head without an outside delivery.

This is the reason that there was no contradiction offered to Murgue's theory for the centrifugal blower as it was for the

centrifugal water pump. In the following analysis, which is practically a summary of the given theories, we will endeavor to formulate the final assumptions to bring about the same conditions for the maximum head in the water pump as it is in the air blower.

We will begin the analysis of the fluid motion inside the pump with zero discharge, the way the water pump is usually started in practice. We may conceive the mass of the fluid between the vanes as a rigid body connected somehow to the center and forced to revolve. Such is the case of a revolving circular plate or solid fly-wheel, each elementary particle containing an amount of energy equal to $\frac{1}{2}m\omega^2r^2 = \frac{1}{2}mV^2$. The summation of this energy is equal to the work done in building up the acquired velocity or to the work required to stop the motion of the revolving mass. The centrifugal force of the revolving

particles is distributed over their respective surfaces as an internal stress, and for a radial element of an angle -de- and height -dr- at a distance r from the center we have the crosssectional area equal to rde-dr, the mass for unit thickness and density y is equal to

$$\frac{y}{g} r d\theta \cdot dr$$

and the centrifugal force per unit of surface is

$$\frac{y}{g} r d\theta . dr . \omega^2 r \div r d\theta$$

$$= \frac{y}{g} \omega^2 r dr \quad . \; . \; . \; . \; . \; . \; . \; (1)$$

We may approach nearer to the actual conditions. The result will not be altered if we disconnect the particles of the mass from the center, and in order to hold them together while revolving about the center we substitute a smooth (frictionless) surface to enclose the region of the vanes. Evidently that surface will experience a pressure per unit of area equal to the summation of the centrifugal forces of the enclosed elementary particles. Integrating equation (1) between the limits r=o and

and r=r we have

$$P = \int_0^r \frac{\gamma}{g}\omega^2 r\,dr = \frac{\gamma}{g}\,\frac{\omega^2 r^2}{2},$$

and the equivalent head

$$\frac{P}{\gamma} = \frac{\omega^2 r^2}{2g} = \frac{V^2}{2g} \quad \cdots\cdots \quad (2)$$

We may now remove the surface and substitute the so called deviating force, which must of course, be equal and opposite to the centrifugal force, or pointed towards the center of rotation.

Since the deviating force is assumed now to be substituted for a frictionless surface, to make up an ideal condition, it follows:

First; each particle of the revolving mass must have a true circular motion, because if any of them shall fail to do so it would mean that the deviating force is not complete, which is contrary to the supposition of an ideal surface.

Second; no energy is required to maintain

a uniform rotation after it is once started (neglecting axle friction as well) since we assume a frictionless condition for all surfaces in contact.

Third; any mass present in the casing outside of the revolving circle is subjected to the pressure experienced by the surface of contact, but is not affected by the motion of the revolving fluid, since that motion is parallel to the surface of contact, and the only effective component could be friction which is neglected in the ideal condition.

This is the ideal condition which may be assumed at zero discharge of the pump, when there is no flow through the vanes, and the solution for the head of impending delivery when friction is neglected is the equivalent of the deviating force equal and opposite to the centrifugal force of the revolving mas which is

$$\frac{V^2}{2g}$$

or the velocity necessary at the start for a given pressure is

$$V=\sqrt{2gH}$$

Now we will continue to analyze the motion of the fluid particles through the pump, running at a velocity V, when the discharge valve is opened. The difference in pressure starts a flow from the casing and the amount discharged is replaced from the supply passing through the vanes.

Let "w" be the weight of a particle discharged from the pump in a short interval of time dt. This particle was originally at rest under a pressure P of the supply tank. During the time dt it was changed from the pressure P to a pressure P existing at the tips of the blades, and also from rest to an absolute velocity V' on leaving the blades. The total energy acquired by the particle is evidently the sum of the two. The first part

is equal to the weight of the particle times the head equivalent to the difference in pressure between the surface of the impeller and the supply tank, or

$$\boxed{wh = w\frac{(P_2 - P_1)}{y}}$$

$(P_1 - P_2)$ being the difference in pressure in pounds per square foot and y the density of the fluid. The second part of energy which is equal to

$$w\frac{V'^2}{2g}$$

can be resolved into two components, one radial equal to $w\frac{V_2^2}{2g}$ where V is the radial velocity at the top of the impeller, and another, tangential equal to $w\frac{u^2}{2g}$, u being the tangential velocity at the same place.

The hydrostatic pressure at the surface of the impeller and the radial component of the absolute velocity of the particles are measured by the centrifugal force of the total mass revolving with the impeller, and are equivalent, as shown previously, to a head of

$$h = \frac{V^2}{2g}$$

where v is the velocity of the tips of the impeller blades. The tangential component of the motion of a particle in intermediate contact with a blade has a maximum value when the end of the blade is radial and u equals v, then its energy is equal to $\frac{V^2}{2g}$ and therefore the same particle will theoretically rise to a height equivalent to the total head

$$\frac{V^2}{2g} + \frac{V^2}{2g}, \text{ or } \frac{V^2}{2g}$$

The following illustration will show more clearly the fact that the amount of work required to discharge a particle of mass from the pump, and therefore the amount of energy contained in it when leaving the blades, is twice that of a particle revolving uniformly with the same speed.

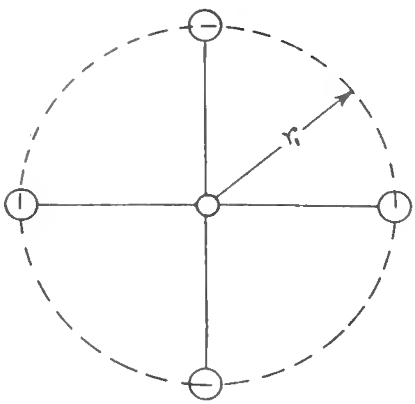

Fig. 1

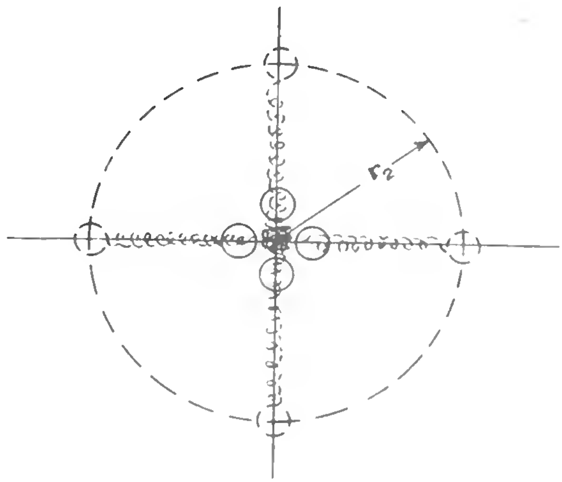

Fig. 2

Figure 1 shows a number of particles rigidly attached at a distance r, from the center, and figure 2 has the same amount of particles, of the same mass attached close to the center by flexible springs. Both systems are brought to a uniform rotation of an angular velocity ω of such a magnitude to stretch out the springs and make r_2 equal to r_1. In the first system the work done on each particle is equal to $\frac{1}{2} m\omega^2 r^2 = \frac{1}{2} m V^2$ is the kinetic energy of the rotation of the particle. In the second system the same amount of work must be done for this part as in the first, and in addition to that the spring offers a resistance R equal and opposite to the centrifugal force of the particles $m\omega^2 r$ varying with the distance from the center. The amount of work required to overcome this variable resistance through the distance r equals to

$$\int_0^r m\omega^2 r\,dr = \frac{1}{2} m\omega^2 r^2 = \frac{1}{2} mv^2$$

Therefore the total work equal to

$$\frac{1}{2}mV^2 + \frac{1}{2}mV^2 = mV^2, \quad \text{or } w\frac{V^2}{g}$$

This is actually what happens with a particle leaving the blades of the pump. It was brought up to a certain pressure and given a velocity at <u>that pressure</u>, and therefore the energy acquired by the particle is necessarily the sum of the two.

If we assume now the water pump to have the outlet open into a stand pipe of sufficient height, we have at a given speed of the blades a condition of flow through the impeller and no discharge from the pump.

In such a case the total kinetic energy acquired by the particles on leaving the blades, which for a unit weight is equal to $\frac{V^2}{g}$, will be converted into potential energy, raising the column in the stand pipe to a theoretical height of

$$H = \frac{V^2}{g}.$$